

Generalized hidden Kerr/CFT

David A. Lowe, Antun Skanata

Brown University, Department of Physics, Box 1843, Providence, RI 02912, USA

Abstract

We construct a family of vector fields that generate local symmetries in the solution space of low frequency massless field perturbations in the general Kerr geometry. This yields a one-parameter family of $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$ algebras. We identify limits in which the $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$ algebra contracts to an $SL(2, \mathbb{R})$ symmetry of the Schwarzschild background. We note that for a particular value of the free parameter, the symmetry algebra generates the quasinormal mode spectrum of a Kerr black hole in the large damping limit, suggesting a connection between the hidden conformal symmetry and a fundamental CFT underlying the quantum Kerr black hole.

1. Introduction

The hidden Kerr/CFT proposal [1] has drawn a good amount of attention since its formulation, and in the past year has been applied to a number of gravitational backgrounds [2, 3, 4, 5, 6]. The allure of the proposal lies in the notion that conformal symmetries need not be manifest symmetries of the geometry in order to consider a conformal field theory (CFT) description of low frequency scattering processes in Kerr background. This should be contrasted with the usual geometric approach where AdS/CFT methods may be applied for near-extremal black holes with throat geometries containing AdS subspaces, or for more general black holes in asymptotically AdS spacetimes.

Low frequency physics in black hole backgrounds has already proved its fruitfulness in [7] by observing that the scalar low energy decay spectrum shows characteristic behavior seen in CFT correlation functions. Similar results have been obtained in [8] and in the presently discussed case of low frequency massless scalar excitations in the near region of the Kerr black hole [1]. The hope is that by studying this low frequency limit, we learn about the underlying conformal field theory conjectured to provide a holographic description of the full quantum Kerr black hole.

In the present work we generalize the hidden CFT generators of [1] to a one-parameter family. For special values of the parameter, a contraction to a single $SL(2, \mathbb{R})$ factor generates symmetries of the Schwarzschild black hole. Here we find agreement with Schwarzschild symmetry generators found in [9]. Moreover, if we assume that the $SL(2, \mathbb{R})$ factors are enhanced to full Virasoro symmetries

underlying the CFT, state counting in the CFT is able to reproduce the exact Kerr entropy. Finally we speculate on the connection between this hidden CFT and a more fundamental CFT describing the black hole. In the large damping limit, hints of CFT structure also emerge [10], and we are able to reproduce the spectrum of the Kerr quasinormal modes from a particular choice of our free parameter, generalizing the result of [9] for Schwarzschild.

2. Scalar field equation

Our analysis will be very much in the spirit of hidden Kerr/CFT proposal [1]. The Kerr metric is given in Boyer-Lindquist coordinates:

$$ds^2 = -(1 - 2Mr/\Sigma) dt^2 - (4Mar \sin^2 \theta / \Sigma) dt d\phi + (\Sigma/\Delta) dr^2 + \Sigma d\theta^2 + \sin^2 \theta (\Delta + 2Mr(r^2 + a^2)/\Sigma) d\phi^2, \quad (1)$$

where $\Sigma = r^2 + a^2 \cos^2 \theta$, $\Delta = r^2 + a^2 - 2Mr$, M is the black hole mass and $J = Ma$ angular momentum. The two horizons are located at $r_{\pm} = M \pm \sqrt{M^2 - a^2}$.

A massless scalar field $\psi(t, r, \theta, \phi)$ propagating in such a background satisfies a Teukolsky [11] wave equation, and with the following ansatz¹

$$\psi \sim e^{im\phi - i\omega t} S(\theta) R(r),$$

the equation can be separated into an angular and radial part:

$$\left[\frac{1}{\sin \theta} \partial_{\theta} (\sin \theta \partial_{\theta}) - \frac{m^2}{\sin^2 \theta} + \omega^2 a^2 \cos^2 \theta + K_l \right] S(\theta) = 0, \quad (2)$$

$$\left[\partial_r (\Delta \partial_r) + \frac{(2Mr_+ \omega - am)^2}{(r - r_+)(r_+ - r_-)} - \frac{(2Mr_- \omega - am)^2}{(r - r_-)(r_+ - r_-)} + (r^2 + 2M(r + 2M))\omega^2 - K_l \right] R(r) = 0. \quad (3)$$

In the low frequency limit $M\omega \ll 1$, the equation (2) reduces to a Laplacian on S^2 , with eigenfunctions being spherical harmonics and eigenvalues $K_l \approx l(l+1)$, $l = 0, 1, \dots$. We will not focus any further on the angular equation (2) and its properties beyond zeroth order solution; the interested reader might want to consult [13, 14] and references within.

For the radial equation (3) we consider a low frequency, near region limit

$$r\omega \ll 1, \quad M\omega \ll 1, \quad (4)$$

following [1]. This allows one to drop the $(r^2 + 2M(r + 2M))\omega^2$ term in (3), at which point the equation reduces to hypergeometric form.

¹The results below generalize in a straightforward way to massless spin 1 and spin 2 wave equations, see for example [12]. Here we restrict our attention to the scalar field case.

If one is interested in Kerr black holes far from extremality, another interesting possibility arises. Namely, one can demand that $r - r_-$ be sufficiently large that the order ω and higher terms coming from the pole near $r \rightarrow r_-$ in (3) be subleading. That is, we may introduce the deformation parameter κ and deform (3) to

$$\left[\partial_r (\Delta \partial_r) + \frac{(2Mr_+ \omega - am)^2}{(r - r_+)(r_+ - r_-)} - \frac{(2M\kappa r_+ \omega - am)^2}{(r - r_-)(r_+ - r_-)} \right] R(r) = l(l+1)R(r), \quad (5)$$

This leaves the low frequency limit unchanged, as long as the two constraints

$$\begin{aligned} \frac{\kappa M^2 am \omega}{(r - r_-)(r_+ - r_-)} &\ll 1, \\ \frac{\kappa^2 M^4 \omega^2}{(r - r_-)(r_+ - r_-)} &\ll 1, \end{aligned} \quad (6)$$

are satisfied. A version of this deformation for the Schwarzschild black hole was considered in [9]. It should be noted that these conditions are implied by the near-region condition (4) as long as $r - r_-$ does not vanish. Thus these conditions are a rather weak modification of the near-region limit.

Let us conclude this section with some additional motivation for the deformation. One might consider simply deforming the positions of the singularities and coefficients in (5) in an arbitrary way, such that the wave equation reduced to (3) as $\omega \rightarrow 0$. However if the coefficient involving the singularity at $r = \infty$ or at $r = r_+$ is deformed, the low energy solutions to the wave equation are changed in a drastic way, since the coefficients control the divergence of the solutions at the singular points. Shifting the positions of these singularities produces a deformation that could only be explained by an action involving more than two time derivatives, which we choose not to consider in the present work. However the inner horizon $r = r_-$ is a special case, because we expect the full nonlinear solution for a perturbation of Kerr to become singular there. Including back-reaction is expected to yield an asymptotically null spacelike singularity capping the would-be inner horizon [15]. Since the low energy linearized wave equation is not relevant at this singular surface near $r = r_-$ it is natural to explore deformations of the wave equation near this point. The κ deformation in (5) is the unique such deformation of the linearized equation of motion, yielding an equation of motion second order in time derivatives.

3. Constructing $SL(2, \mathbb{R})$

We consider a set of vector fields that satisfy an $SL(2, \mathbb{R})$ algebra. We also demand the quadratic Casimir reproduces the scalar field wave equation in the Kerr background in the near region low frequency approximation (4) subject to the additional constraints (6). One general form of such vector fields is

$$\begin{aligned}
L_{\pm} &= e^{\pm\alpha t \pm \beta \phi} (g_{\pm}(r)\partial_r + h_{\pm}(r)\partial_{\phi} + k_{\pm}(r)\partial_t), \\
L_0 &= \gamma\partial_t + \delta\partial_{\phi}.
\end{aligned} \tag{7}$$

The requirement that L_0 is an eigenvector of a state $\psi \sim e^{im\phi - i\omega t} R(r)S(\theta)$ sets γ and δ to constants. The constraints we impose are:

$$\begin{aligned}
[L_+, L_-] &= 2L_0, \\
[L_{\pm}, L_0] &= \pm L_{\pm}, \\
L_0^2 - \frac{1}{2}(L_+L_- + L_-L_+) &= \partial_r(\Delta\partial_r) + f(r),
\end{aligned}$$

where $f(r)$ is a function that involves no single derivatives in t or ϕ , except for $\partial_t\partial_{\phi}$. Given these constraints, we claim the following is the most general functional form of such generators:

$$\begin{aligned}
L_{\pm} &= e^{\pm\alpha t \pm \beta \phi} \left(\mp\sqrt{\Delta}\partial_r + \frac{C_2 - \delta r}{\sqrt{\Delta}}\partial_{\phi} + \frac{C_1 - \gamma r}{\sqrt{\Delta}}\partial_t \right), \\
L_0 &= \gamma\partial_t + \delta\partial_{\phi},
\end{aligned} \tag{8}$$

with constraints on parameters arising from imposing the $sl(2, \mathbb{R})$ algebra:

$$\begin{aligned}
\alpha C_1 + \beta C_2 + M &= 0, \\
1 + \alpha\gamma + \beta\delta &= 0.
\end{aligned} \tag{9}$$

These determine α and β . The last three equations we impose are the ones identifying appropriate terms in the quadratic Casimir with ∂_{ϕ}^2 , $\partial_t\partial_{\phi}$ and ∂_t^2 terms in the wave equation (5). The ∂_{ϕ}^2 term gives us the branches:

$$\begin{aligned}
\delta &= \pm a/\sqrt{M^2 - a^2} & \delta &= 0 \\
C_2 &= M\delta & C_2 &= \pm a.
\end{aligned}$$

The differing signs simply generate automorphisms of the algebra, so may be dropped in the following. Examining the two remaining terms gives:

$$\gamma\delta a^2 - C_1C_2 - r(2M\gamma\delta - C_2\gamma - C_1\delta) = -\frac{2Mr+a}{r_+ - r_-} [r(1 - \kappa) - (r_- - \kappa r_+)],$$

and

$$\gamma^2 a^2 - C_1^2 - 2r\gamma(M\gamma - C_1) = -\frac{4M^2 r_+^2}{r_+ - r_-} [r(1 - \kappa^2) - (r_- - \kappa^2 r_+)].$$

The two possible branches are shown in Table 1 describing a one-parameter family of $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$ generators labelled by κ .

$\gamma = \frac{2Mr_+}{r_+ - r_-}(\kappa + 1)$ $\delta = \frac{2a}{r_+ - r_-}$ $C_1 = \frac{2Mr_+}{r_+ - r_-}(\kappa r_+ + r_-)$ $C_2 = M\delta$	$\gamma = \frac{2Mr_+}{r_+ - r_-}(\kappa - 1)$ $\delta = 0$ $C_1 = \frac{2Mr_+}{r_+ - r_-}(\kappa r_+ - r_-)$ $C_2 = a$
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Table 1: Two branches of solutions for the generators.

The solution for the generators is

$$\begin{aligned}
L_{\pm} &= e^{\mp 2\pi T_R \phi} \left[\mp \sqrt{\Delta} \partial_r - \frac{1}{2\pi T_H} \frac{r-M}{\sqrt{\Delta}} (\Omega \partial_{\phi} + \partial_t) \right. \\
&\quad \left. + \frac{1}{2\pi \Omega(T_L + T_R)} \frac{r-r_+}{\sqrt{\Delta}} \partial_t \right] \\
L_0 &= \frac{1}{2\pi T_H} (\Omega \partial_{\phi} + \partial_t) - \frac{1}{2\pi \Omega(T_L + T_R)} \partial_t
\end{aligned} \tag{10}$$

and

$$\begin{aligned}
\bar{L}_{\pm} &= e^{\pm 2\pi \Omega(T_L + T_R)t \mp 2\pi T_L \phi} \left[\mp \sqrt{\Delta} \partial_r + \frac{2Mr_+}{\sqrt{\Delta}} (\Omega \partial_{\phi} + \partial_t) \right. \\
&\quad \left. + \frac{1}{2\pi \Omega(T_L + T_R)} \frac{r-r_+}{\sqrt{\Delta}} \partial_t \right], \\
\bar{L}_0 &= -\frac{1}{2\pi \Omega(T_L + T_R)} \partial_t,
\end{aligned} \tag{11}$$

where $T_H = \frac{\sqrt{M^2 - a^2}}{4\pi M r_+}$ is Hawking temperature and $\Omega = \frac{a}{2Mr_+}$ angular velocity at the outer horizon, and we have introduced ‘‘CFT’’ temperatures as

$$T_R = \frac{\sqrt{M^2 - a^2}}{2\pi a} \tag{12}$$

and

$$T_L = T_R \frac{1 + \kappa}{1 - \kappa}. \tag{13}$$

We note the generators (11) shift the frequency of a mode by the imaginary amount $2\pi \Omega(T_L + T_R)$. This means we only stay within the low frequency limit (4) provided we are close to the extremal Kerr limit, so that $T_R \ll 1$ (assuming κ is fixed). Outside this limit the descendants of some primary operator are no longer mapped to eigenfunctions of the low frequency scalar field equation. The other set of generators (10) do not suffer from this additional constraint.

Higher order corrections to the Teukolsky equation in the low frequency limit give rise to soft breaking of the conformal symmetry, and running of the anomalous dimensions, as described in [12].

In addition, global identifications on the solution space by $\phi \rightarrow \phi + 2\pi$ explicitly breaks the symmetry algebra down to $U(1) \times U(1)$ generated by (L_0, \bar{L}_0) .

Finally let us comment on two special cases where the general solutions (10) and (11) do not hold. For $\kappa = 1$ and general rotation parameter a , the right branch in Table 1 fails to yield a consistent solution to the constraint equations, so only the left branch generates an $SL(2, \mathbb{R})$. Since we only find one $SL(2, \mathbb{R})$ for general a we are unable to carry through the conjecture that the theory should be dual to a 2d CFT, so we do not pursue this case further in this paper.

The Schwarzschild case, $a = 0$, should likewise be treated as a special case. Here we find the constraint equations are inconsistent unless $\kappa = \pm 1$. For both values of κ the same single copy of $SL(2, \mathbb{R})$ is found. This may be read off, for example, from the right branch of Table 1 by setting $\kappa = -1$ and $a = 0$.

4. Relation to known results

If we set $\kappa = r_-/r_+$ our results match those of [1]. For this choice of κ the pole term in (3) is exact, so the subsidiary constraints (6) may be dropped.

It is also interesting to consider the Schwarzschild limit, where our results match those of Bertini, Cacciatori and Klemm [9]. They find a single set of $SL(2, \mathbb{R})_{Sch}$ generators

$$L_{\pm} = e^{\pm t/4M} \left(\pm \sqrt{\Delta} \partial_r - \frac{4M(r-M)}{\sqrt{\Delta}} \partial_t \right), L_0 = -4M \partial_t. \quad (14)$$

The generators constructed at the end of Section 3 for $a = 0$ and $\kappa = -1$ match these, up to the automorphism $L_{\pm} \rightarrow -L_{\pm}$ and $L_0 \rightarrow L_0$.

5. Quasinormal modes

It is well known that classical black holes are characterized by a discrete set of complex frequencies, named quasinormal modes. The quasinormal modes correspond to a certain set of boundary conditions, with waves purely outgoing at infinity and ingoing at the horizon. Two observations immediately jump to mind: a quantum theory of gravity should reproduce this spectrum; and if the states in this quantum theory are fully characterized by quasinormal modes, studying semiclassical physics outside the black hole horizon should teach us about this quantum theory.

The connection between quasinormal modes for three-dimensional black holes and CFT states has been made precise in [16] by looking at linearized perturbations² and showing an explicit agreement between quasinormal frequencies and the poles of the retarded correlation function in the CFT.

Likewise for the Kerr black hole, there is a discrete spectrum of quasinormal modes [17]. At large damping the imaginary part of the frequency increases

²The previous definition of quasinormal modes via ingoing flux at infinity does not make it if we put the system in a box. The way quasinormal modes were defined in asymptotically AdS backgrounds was to impose either Dirichlet boundary conditions at asymptotic infinity, or a vanishing flux $\mathcal{F} \sim \sqrt{-g} (R^* \partial_\mu R - c.c.)$. Both choices lead to same spectrum.

approximately linearly with mode number, while the real part approaches a constant. According to Keshet and Neitzke [10], large damping is where one expects a CFT description to emerge, as the transmission and reflection amplitudes take a familiar CFT-like form. Moreover, a step towards this understanding has been made in [18], by obtaining the quasinormal mode spectrum via a WKB approximation to the wave equation.

An interesting observation was made in [9] – the descendant states $(L_-)^n \psi(t, r, \phi)$ reproduce the large damping quasinormal spectrum of the Schwarzschild black hole. We speculate this might be the case with generators (11) as well, giving a connection between the low frequency hidden CFT, and some more fundamental underlying CFT that correctly describes the quasinormal modes.

Keshet and Hod [18] compute the quasinormal mode spectrum at large damping and obtain

$$\omega = -m\hat{\omega} - 2\pi i T_0(n + 1/2), \quad (15)$$

to leading order in n , where $T_0 = f(a/M)T_H$ is a smooth slowly varying function of angular momentum with $f(0) = 1$, that may be expressed in general using elliptic integrals.

We can choose the value of $\kappa(a)$ by solving

$$T_0 = \Omega T_R \frac{2}{1 - \kappa}.$$

Then by defining the lowest weight state via

$$\begin{aligned} \bar{L}_0 \Phi^{(0)} &= \bar{h} \Phi^{(0)}, \\ \bar{L}_+ \Phi^{(0)} &= 0, \end{aligned}$$

it is easy to check the descendants $\Phi^{(n)} = (\bar{L}_-)^n \Phi^{(0)}$ reproduce the large n behavior of the spectrum (15).

6. Entropy

Following the discussion of the introduction, we can propose that the $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$ symmetry is promoted to a full left and right-moving Virasoro symmetry in the full quantum theory. The Cardy formula gives

$$S = \frac{\pi^2}{3} (c_L T_L + c_R T_R). \quad (16)$$

The T_L and T_R appearing in (12) and (13) may be matched with the left and right-moving CFT temperatures. The CFT inherits periodic identifications in the imaginary left and right moving directions, from the periodic identification of ϕ , and the action of the Virasoro generators.

The Bekenstein-Hawking entropy is

$$S_{BH} = 2\pi M r_+, \quad (17)$$

which agrees exactly with (16) provided we identify the central charge as

$$c_L = c_R = \frac{6a(1 - \kappa)Mr_+}{\sqrt{M^2 - a^2}}. \quad (18)$$

We emphasize the identification of the central charge (18) is not an independent computation. Such a computation may well be possible, but would require a reworking of the Brown-Henneaux calculation [19] within the hidden CFT/low frequency framework.

The central charge depends in a nontrivial way on the deformation parameter κ . This indicates the dual description is actually a family of conformal field theories with a conformal deformation parameter. While we do not yet have enough data to specify the CFTs at hand in detail, there are many examples of such families of conformal field theories. A simple example is the level number of a conformal field theory based on an affine Lie algebra. A much more general set of examples, including continuous deformations of CFTs that change the central charge, appears in [20].

The formula (18) reduces to the result of [1] when $\kappa = r_-/r_+$ where $c_L = c_R = 12J$. There one can argue that in the extremal limit, the central charge follows from standard geometric argument [21]. However a closer look at this shows the low frequency symmetry generators of [1] do not smoothly match the isometry generators of [21]. Thus the fixed point of the low frequency conformal symmetry of [1] does not coincide with extremal Kerr [12]. Moreover a strong argument for the non-renormalization of the central charges away from the extremal point is lacking for the case considered in [1]. The addition of the extra parameter κ in (18) does not help this situation.

In the Schwarzschild limit we only retain a single $SL(2, \mathbb{R})$ symmetry, which might be associated with a conformal quantum mechanics. If we assume this alone is promoted to a full Virasoro symmetry, we find periodicity with respect to ϕ no longer fixes the CFT temperature. Rather we must return to the generators (14), where we can read off the temperature

$$T_{CFT} = T_H = \frac{1}{8\pi M}.$$

The central charge for conformal quantum mechanics dual to Schwarzschild is then predicted to be

$$c_{Sch} = 96M^3, \quad (19)$$

an apparently new result.

7. Conclusions

We have obtained a generalization of the hidden low frequency limit of the wave equation in a general Kerr background. This hints at an underlying conformal field theory description, that we have explored. The most exciting aspect of this work is the idea that the underlying hidden conformal field theory description may be studied via low frequency scattering, a kind of nonlocal probe

of the geometry, rather than simply looking for geometric isometries as has been already much studied in the literature. This opens up the possibility that even the Schwarzschild black hole may have a CFT dual.

To put these results on a firmer footing, it would be interesting to obtain the central extensions (18) and (19) from some generalization of the asymptotic symmetry algebra analysis of [19]. We hope to explore this issue in future work.

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References

- [1] A. Castro, A. Maloney, A. Strominger, Hidden Conformal Symmetry of the Kerr Black Hole, *Phys. Rev. D* **82** (2010) 024008. [arXiv:1004.0996](#), [doi:10.1103/PhysRevD.82.024008](#).
- [2] R. Fareghbal, Hidden Conformal Symmetry of Warped AdS_3 Black Holes, *Phys. Lett. B* **694** (2010) 138–142. [arXiv:1006.4034](#), [doi:10.1016/j.physletb.2010.09.043](#).
- [3] C. Krishnan, Hidden Conformal Symmetries of Five-Dimensional Black Holes, *JHEP* **07** (2010) 039. [arXiv:1004.3537](#), [doi:10.1007/JHEP07\(2010\)039](#).
- [4] M. Cvetič, F. Larsen, Conformal Symmetry for General Black Holes, [arXiv:1106.3341](#)[arXiv:1106.3341](#).
- [5] M. Cvetič, F. Larsen, Conformal Symmetry for Black Holes in Four Dimensions, [arXiv:1112.4846](#).
- [6] D. Anninos, S. A. Hartnoll, D. M. Hofman, Static Patch Solipsism: Conformal Symmetry of the de Sitter Worldline, [arXiv:1109.4942](#)[arXiv:1109.4942](#).
- [7] J. M. Maldacena, A. Strominger, Universal low-energy dynamics for rotating black holes, *Phys. Rev. D* **56** (1997) 4975–4983. [arXiv:hep-th/9702015](#), [doi:10.1103/PhysRevD.56.4975](#).
- [8] M. Cvetič, F. Larsen, Greybody factors for rotating black holes in four dimensions, *Nucl. Phys. B* **506** (1997) 107–120. [arXiv:hep-th/9706071](#), [doi:10.1016/S0550-3213\(97\)00541-5](#).
- [9] S. Bertini, S. L. Cacciatori, D. Klemm, Conformal structure of the Schwarzschild black hole, *Phys. Rev. D* **85** (2012) 064018, 17 pages, uses JHEP3.cls. v2: Minor correction in appendix. v3: Final version to appear in PRD. [arXiv:1106.0999](#), [doi:10.1103/PhysRevD.85.064018](#).
- [10] U. Keshet, A. Neitzke, Asymptotic Spectroscopy of Rotating Black Holes, *Phys. Rev. D* **78** (2008) 044006. [arXiv:0709.1532](#), [doi:10.1103/PhysRevD.78.044006](#).

- [11] S. A. Teukolsky, Rotating black holes - separable wave equations for gravitational and electromagnetic perturbations, *Phys. Rev. Lett.* 29 (1972) 1114–1118. doi:10.1103/PhysRevLett.29.1114.
- [12] D. A. Lowe, I. Messamah, A. Skanata, Scaling dimensions in hidden Kerr/CFT, *Phys. Rev. D* 84 (2011) 024030. arXiv:1105.2035, doi:10.1103/PhysRevD.84.024030.
- [13] E. D. Fackerell, R. G. Crossman, Spin - weighted angular spheroidal functions, *J. Math. Phys.* 18 (1977) 1849.
- [14] E. Berti, V. Cardoso, M. Casals, Eigenvalues and eigenfunctions of spin-weighted spheroidal harmonics in four and higher dimensions, *Phys. Rev. D* 73 (2006) 024013. arXiv:gr-qc/0511111, doi:10.1103/PhysRevD.73.024013.
- [15] P. R. Brady, S. Droz, S. M. Morsink, The Late time singularity inside nonspherical black holes, *Phys. Rev. D* 58 (1998) 084034. arXiv:gr-qc/9805008, doi:10.1103/PhysRevD.58.084034.
- [16] D. Birmingham, I. Sachs, S. N. Solodukhin, Conformal field theory interpretation of black hole quasi-normal modes, *Phys. Rev. Lett.* 88 (2002) 151301. arXiv:hep-th/0112055, doi:10.1103/PhysRevLett.88.151301.
- [17] E. Berti, V. Cardoso, S. Yoshida, Highly Damped Quasinormal Modes of Kerr Black Holes: A Complete Numerical Investigation, *Phys. Rev. D* 69 (2004) 124018. arXiv:gr-qc/0401052, doi:10.1103/PhysRevD.69.124018.
- [18] U. Keshet, S. Hod, Analytic Study of Rotating Black-Hole Quasinormal Modes, *Phys. Rev. D* 76 (2007) 061501. arXiv:0705.1179, doi:10.1103/PhysRevD.76.061501.
- [19] J. D. Brown, M. Henneaux, Central Charges in the Canonical Realization of Asymptotic Symmetries: An Example from Three-Dimensional Gravity, *Commun. Math. Phys.* 104 (1986) 207–226. doi:10.1007/BF01211590.
- [20] J. Freericks, M. Halpern, Conformal Deformation By The Currents Of Affine g , *Annals Phys.* 188 (1988) 258. doi:10.1016/0003-4916(88)90103-0, 10.1016/0003-4916(88)90103-0.
- [21] M. Guica, T. Hartman, W. Song, A. Strominger, The Kerr/CFT Correspondence, *Phys. Rev. D* 80 (2009) 124008. arXiv:0809.4266, doi:10.1103/PhysRevD.80.124008.